# The History of Decision under Risk and Ambiguity, Resulting in the Modern Behavioral Approach

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How the maths in decision theory were dictated to us by data.

Subtitle: How modern developments (on imprecise probabilities = ambiguity) show that mankind did not learn from history …

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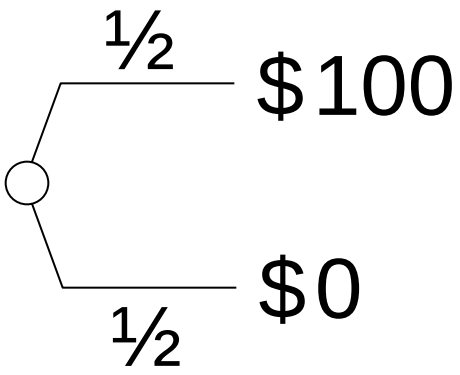
"Those who don't study history are doomed to repeat it. Yet those who *do* study history are doomed to stand by helplessly while everyone else repeats it."

This lecture is on:

- history of risk theory
- analogies with current imprecise ("unknown") probabilities
- speculations on future of field.

Typical  of decision theory: role of empirical findings.  Modern behavioral approach.

# Part I: The history of modeling risk attitude




or $50 for sure

What would you rather have?

Such gambles occur in:

- Public lotteries, casinos, horse races;
- Investments, insurance, medical treatments, etc.;
- Leaving your labtop unattended during lunch in Amsterdam.

Two questions/lines:

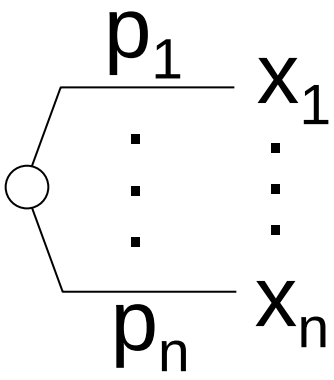
1) How generally model risk attitude?
   To what extent through:
   - sensitivity towards outcomes (utility)
     versus
   - sensitivity towards chance
     (probability weighting)?

2) Prevailing empirical patterns of risk attitude?
   Is risk-aversion
   - universal (modulo noise);
   - systematically violated?

Point 2 will lead to new maths.

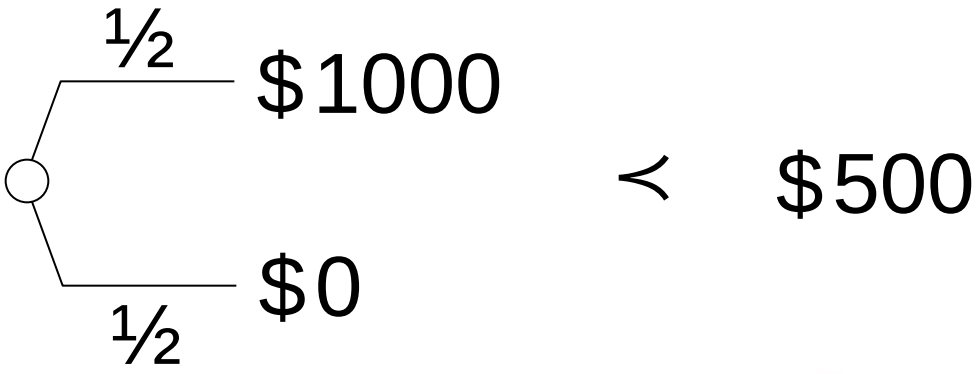
# Simplest way to evaluate risk:

Expected value (Christiaan Huygens 1657)




$$\mapsto \; p_1 x_1 + \cdots + p_n x_n$$

# However, empirical observations:

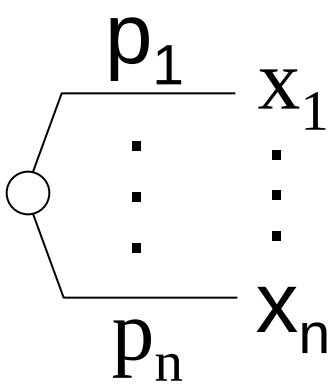



Risk aversion!


Falsifies expected value.

To explain falsifications:
“expected utility” (EU; Bernoulli 1738).


Bernoulli (1738)



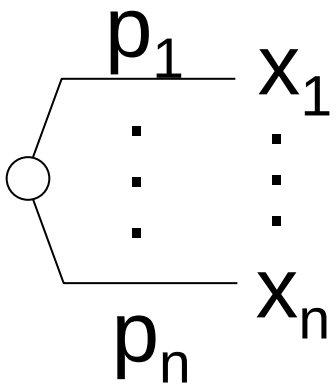

$$\mapsto \; p_1 U(x_1) + \cdots + p_n U(x_n)$$

Big conceptual step: departure from objectivity.

U: subjective index of risk attitude.

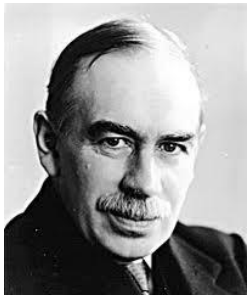
For argument coming next:
easiest to understand for novices.

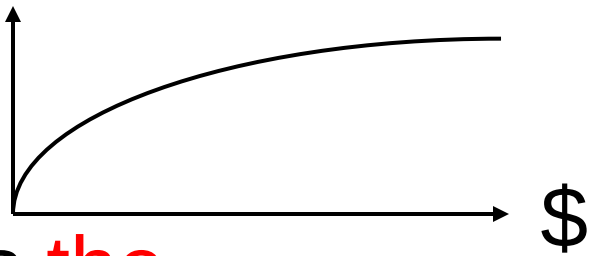
# Risk aversion in general:




$$\preccurlyeq \quad p_1x_1 + \cdots + p_nx_n$$

**Theorem** (Marshall 1890). Risk aversion holds if and only if utility U is concave.






Illustrates how U is used as the subjective index of risk attitude.

Measure of risk aversion: $-U''/U'$ (Pratt & Arrow).

Other often-used index of risk aversion: $-\alpha U''/U'$.

# Line (1) of this talk:

the general modeling of risk attitude.

$\psi^{\underline{s}}$ objected from the beginning:

U

=

sensitivity towards money

≠

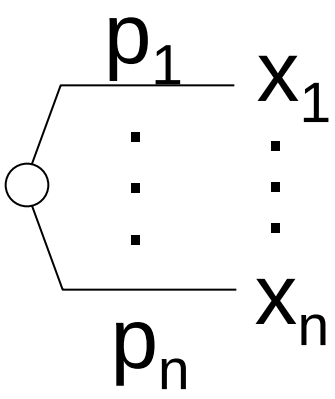
risk attitude.

Theoreticians dislike such “unfounded” reasoning (about processes).

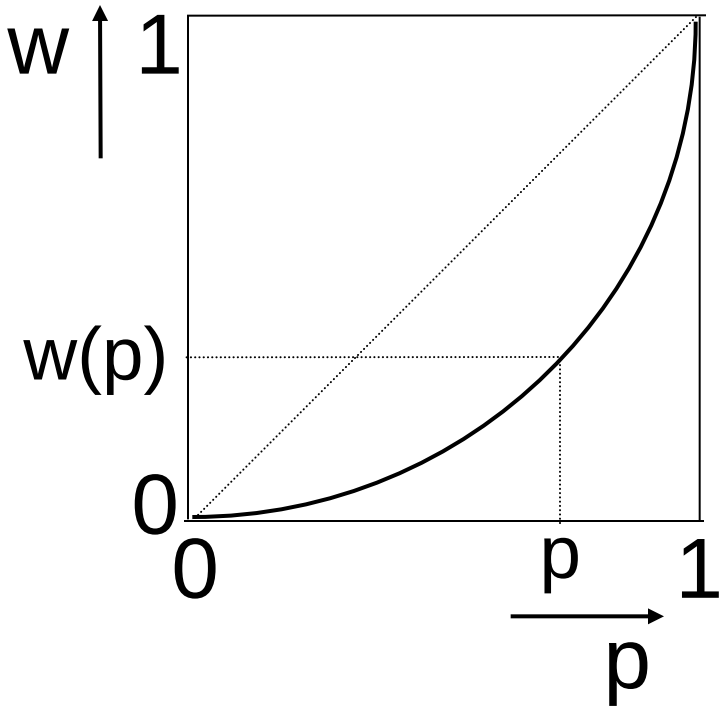
But here it is useful.

# Intuition (primarily from $\psi^{S}$):

Use sensitivity towards probabilities!




$\mapsto \quad w(p_1)U(x_1) \; + \; \cdots \; + \; w(p_n)U(x_n)$




w(0) = 0,
w(1) = 1,
w is increasing.

# Lola Lopes (1987):
# "Risk attitude is more than the psychophysics of money."

utility

The idea (that also probability weighting) had been around before: D'Alembert (1768) "*Opuscules Mathématiques*, vol. iv., (extraits de lettres)":

It seemed to me [in reading Bernoulli's Ars Conjectandi] that this material needs to be treated more clearly; I saw well that the expectation is larger, $1^0$ that the expected sum is larger, $2^0$ that the probability of winning is so too. But I did not see the same evidence, and I still do not see, $1^0$ that the probability were estimated exactly by the methods used; $2^0$ that if it were, the expectation should be proportional to that simple probability, rather than to a power or even to a function of that probability; $3^0$ that if there are several combinations that give different averages or different risks (which one considers as negative averages) one had to be satisfied to simply *add* together all these expectations for having the total expectation." [italics from the original]

Edwards (1950s) studied probability weighting.

$\psi$'s argument is intuitive, not theoretical.

Theoreticians:

"Such arguments are invalid!"

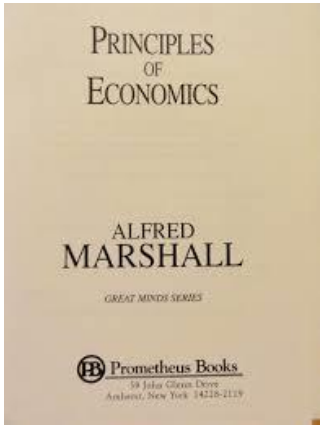
These "experimental theories" never became big.

# Line (2): is risk aversion universal?

## Theoretical arguments (common among economists):

1) diminishing marginal utility is intuitively plausible;
2) concave utility is needed for existence of equilibria;
3) no concave U $\Rightarrow$ market for lotteries.

Marshall, A. (1920)
*Principles of Economics*




about risk-seeking individuals:

... since experience shows that they are likely to engender a restless, feverish character, unsuited for steady work as well as for the higher and more solid pleasures of life.

Typical of these theoretical arguments:
No reference to data at all!

# Problem:

Public lotteries!?!?

Arrow (1971, p.90), about co-existence gambling/insurance:

"I will not dwell on this point extensively, emulating rather the preacher, who, expounding a subtle theological point to his congregation, frankly stated:

> Brethren, here there is a great difficulty; let us face it firmly and pass on."

Experimentalists: ????? They recognized deviations from risk aversion from the beginning (1948).

# Back to line (1), the general modelling of risk attitude.

End of 1970s:
renewed interest
in probability weighting,
a.o. because of violations of EU
(Machina '82).

- Handa (1978, *J. of Pol. Econy*),
- Kahneman & Tversky (1979, *Econmetrica*, "prospect theory").

Prominent economic journals ... !

On Handa (1978), JPE received 10 comments.

Fishburn's (1978) was published.

(Unknown-Australian-Quiggin's wasn't.)

K&T's '79 prospect theory is exceptional success;

2nd most cited economic paper!

But, has theoretical problem.

K&T's (& all then-used) probability weighting violates stochastic dominance!

Amazing that that model could survive in the experimental literature for 30 years ...

Theoreticians - Experimentalists: 1 - 0

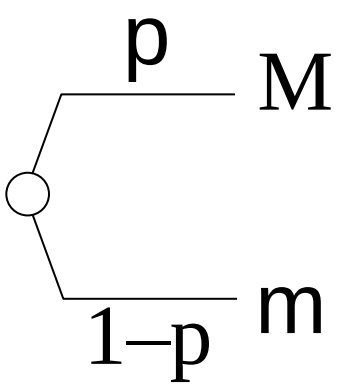
Yet,
"risk-attitude through probability weighting"
is a good intuition.

Only, one should weight the "right" probabilities.

Not
probability of a separate outcome,
but
goodnews probability:
probability of
receiving something better than some outcome.

Right

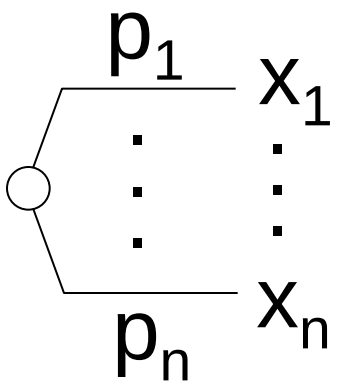
~~Wrong~~ formula for two outcomes $M > m \geq 0$:




$$\mapsto \; w(p)u(M) + \cancel{w(1-p)}\,u(m)$$

$$(1 - w(p))$$

# Evaluation of general lottery



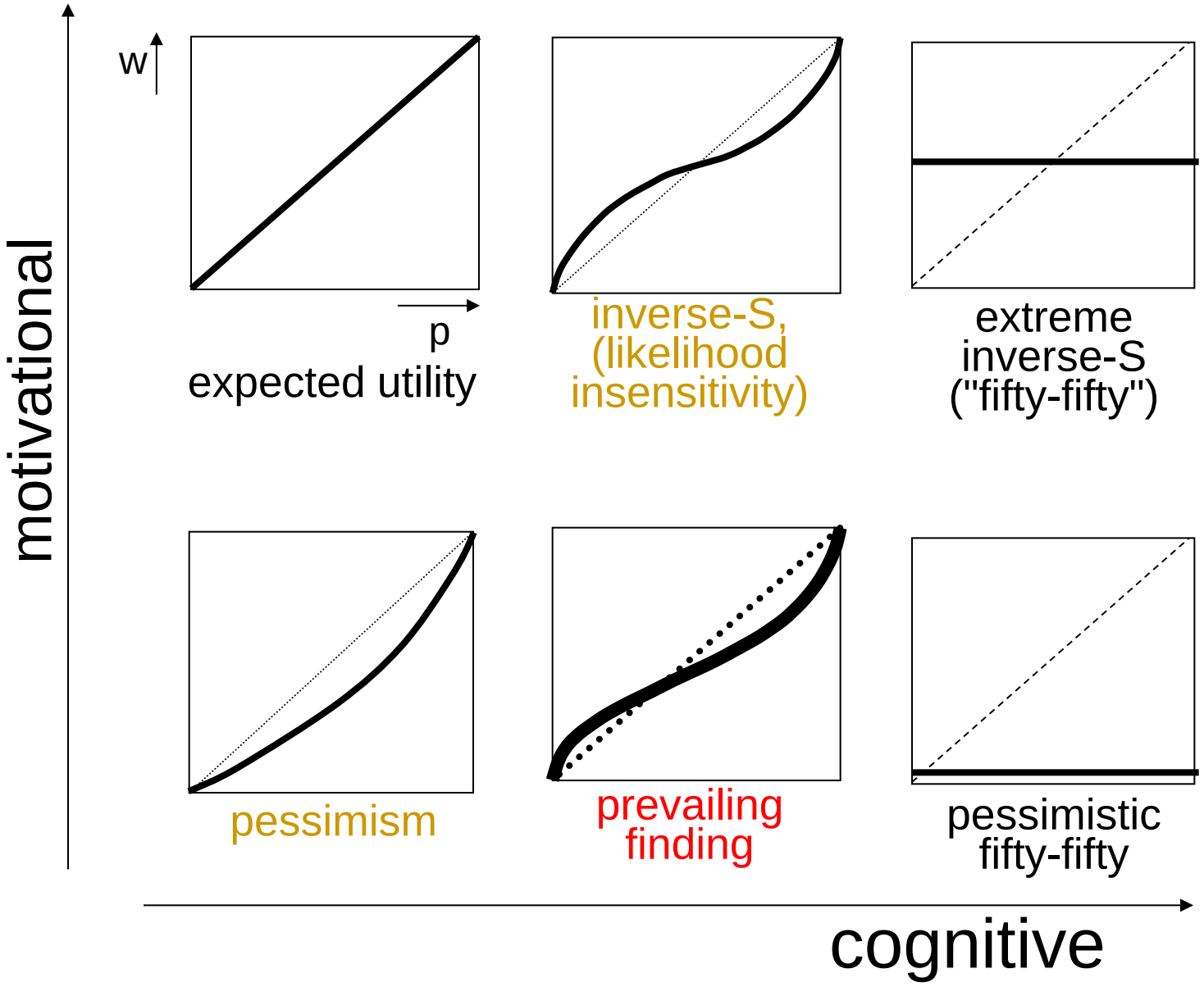

… we skip.

This is the idea of Quiggin (1982), for risk:
Rank-Dependent Utility.

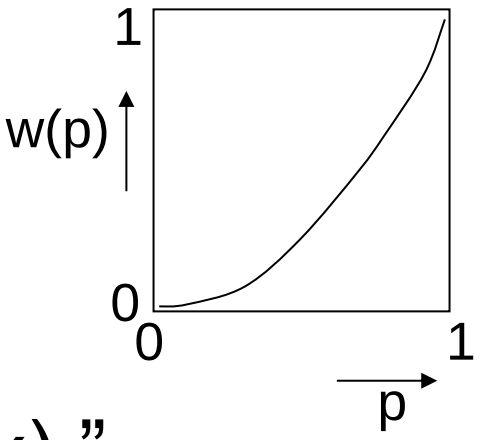
(P.s.: Essentially the same idea for the more subtle ambiguity, independently by Schmeidler (1989).)

Back to line 2, phenomena/risk aversion.
Now we consider the new component, w.
(Similar phenomena will be relevant for imprecise probabilities.)

# Typical shapes of probability weighting



In the beginning, theoretical views:




“Risk-aversion is universal.

U is concave and
prob. weighting w is similar (convex).”

Economists need this to get equilibria.

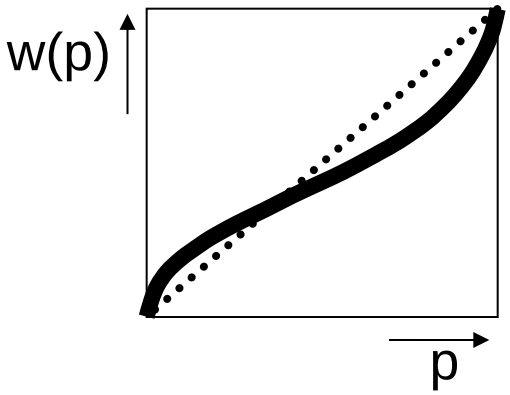
Convex optimization crowd needs it to keep demand for their techniques.

New impulses came from experimental investigations by $\psi^{\underline{s}}$ (Tversky and others).

# Systematic risk-seeking for:

Small chances at large gains;

large chances at small losses.




prevailing finding

Amazing, that “universal” risk aversion could survive in the theoretical literature for 30 years

…

Theoreticians - experimenters: 1 - 1

# Synthesis for risk:

Tversky, A. & D. Kahneman (1992), "Advances in Prospect Theory: Cumulative Representation of Uncertainty," Journal of Risk and Uncertainty 5, 297 – 323.

New prospect theory (1992):
Risk-attitudes in terms of
- utilities ánd
- probability weighting
(- and loss aversion).

Risk-aversion prevailing,
but, systematic deviations.

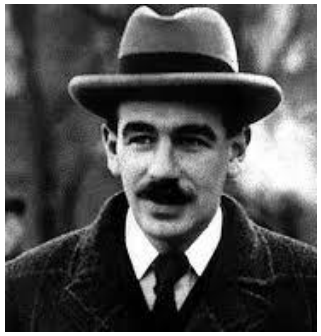
Reference point (“framing”).

Theory combines
- descriptive power of ’79 prospect theory
- theoretical power of economic theories.

Kahneman & Tversky (1979) original prospect theory:
birth of behavioral decision theory.
Tversky & Kahneman (1992) new prospect theory:
behavioral decision theory became mature.
It also handles imprecise probabilities.
Comes next.

# Part II. The (history and) future of imprecise probabilities (ambiguity)

## Keynes & Knight (1921):



Real uncertainty if: new risks.
Unique events; not seen before.
No statistics. No averaging out.

For example, financial crises are always due to unforeseen, new, events. No statistics. No hedges.

In many strategic situations:

no such opponent before.

Insurance for big rare catastrophes: nope …

# New uncertainties: "imprecise probabilities" (in economics: "ambiguity")

Ubiquitous in business & economics.
Repeatable experiments are not possible with our economy.

Requires new models.
Is behavioral: ambiguity $\Rightarrow$ strong deviations from classical rationality.
Homo sapiens $\neq$ homo economicus.

# First answer to Keynes'21 & Knight'21
(how handle imprecise probabilities?)
by Ramsey'31, de Finetti'31, Savage'54:



Always, also if no precise probabilities,
then still continue to use probabilities, being
subjective probabilities!

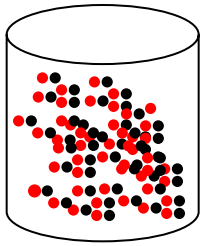
Ellsberg's (1961) paradox:
Subjective probabilities don't work
(at least not in classical sense).

Here comes his paradox:

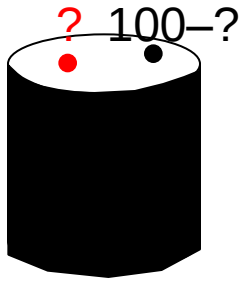
# Ellsberg paradox

Known urn K | Ambiguous urn A

50 R
50 B



100 R&B
in unknown
proportion




$(R_K: €15) \succ (R_A: €15)$

$(B_K: €15) \succ (B_A: €15)$

$P(R_K) > P(R_A)$

$P(B_K) > P(B_A)$

$\frac{}{1} + \succ \frac{}{1} +$

Violates subjective probabilities:

Violates subjective probabilities.

Subjective probabilities do not work.

Since 1921/1961:
we need something fundamentally new.

Only in late 1980s, people clever enough
to invent something fundamentally new:

Gilboa & Schmeidler (‘87, ’89).
This explains:
- why imprecise probabilities, while important, took off only late 1980s;
- we have much to catch up;
- imprecise probabilities are so popular today.

# Current state of the art of imprecise probability theory in economics:

Ellsberg urn ➔ ambiguity aversion.
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Camerer & Weber (1992): “There are decreasing returns to studying urns.”

Theoreticians:
take imprecise probabilities as one-dim. thing;
take aversion as the only phenomenon.
Capture attitude in one number:
one index of ambiguity aversion.

But imprecise probs are too rich.
One index of ambiguity aversion
is like
one index of risk aversion for all nonmonetary outcomes.
Ulam (in another context):
“Using a term like nonlinear science is like referring to the bulk of zoology as the study of non-elephant animals.”

# Many theories try to model imprecise probs through utility curvature today (line 1 …):

Klibanoff, Peter, Massimo Marinacci, & Sujoy Mukerji (2005), "A Smooth Model of Decision Making under Ambiguity," *Econmetrica* 73, 1849-1892.

Chew, Soo Hong, King King Li, Robin Chark, & Songfa Zhong (2008), "Source Preference and Ambiguity Aversion: …," *Advances in Health Economics and Health Services Research* 20, 179–201.

Criticized by Epstein (2010 Ectra) and Baillon, Driesen, & Wakker (2012 GEB).

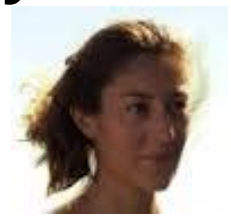
$\psi^{s}$ (Tversky et al.):
Not one attitude towards imprecise probs.
Distinguish between sources of uncertainty;
attitude towards imprecise probs is source-dependent.
(Can be compared to utility:
utility is commodity-dependent.)

As there is much risk seeking, there also is much ambiguity seeking (Einhorn & Hogarth'85).

(line 2:) Amazing, that "universal" ambiguity aversion could survive in the theoretical literature for > 20 years (1990 – 2015) …

Imprecise probs: Before thinking about the weighing of beliefs, have to think:
what those beliefs are.

I like to use PT for ambiguity through the source method, introduced by Abdellaoui, Baillon, Placido, & Wakker (AER 2011).

It centers out one pignistic probability …
New developments: … to come.

# Summary:

1. Classical theory: Expected utility; Risk attitude = U($) (Bernoulli 1738, Marshall 1890).
2. Experimentalists: risk attitude also $\Leftarrow$ w(p) (Edwards, 1954). Took wrong probabilities.
3. Theoreticians:

   Take right ("cumulative") p's (Quiggin, 1981).
   Thought universal risk aversion; convex/concave.
4. Experiments: diminishing sensitivity iso risk aversion (Tversky & Kahneman, 1992); inverse-S.
5. Imprecise probabilities: history is repeated, regarding both lines 1 and 2.
6. Synthesis: New prospect theory

# The end.